

A SEMANTIC APPROACH TO NON-MONOTONIC ENTAILMENTS

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Any inferential system in which the addition of new premises can lead to the retraction of previous conclusions is a non-monotonic logic. Classical conditional probability provides the oldest and most widely respected example of non-monotonic inference.

This paper presents a semantic theory for a unified approach to qualitative and quantitative non-monotonic logic. The qualitative logic is unlike most other non-monotonic logics developed for AI systems. It is closely related to classical (i.e., Bayesian) probability theory. The semantic theory for qualitative non-monotonic entailments extends in a straightforward way to a semantic theory for quantitative partial entailment relations, and these relations turn out to be the classical probability functions.

1. OVERVIEW

Formal logics for AI systems are usually implemented as a controlled sequence of syntactic transformations on expressions in a formal language. The syntactic transformations are designed to compute some underlying notion. -- e.g., some notion of logical entailment, logical consistency, or justified degree of certainty. Ideally the underlying logical notion is made precise by a semantic theory. The semantic theory identifies certain primitive semantic concepts (e.g., truth or satisfaction), and defines more complex semantic concepts in terms of the primitives (e.g., consistency and logical entailment). It provides for the establishment of important semantic theorems (e.g., that some collection of syntactic transformations is truth preserving).

This ideal is well illustrated by automated systems for sentential logic. The underlying semantic theory takes the notion of a truth-value assignment to every sentence as primitive. Semantic rules govern how truth-values may be assigned to complex sentences in terms of the truth-values of constituents. Logical entailment is defined. A sentence A is a logical consequence of B just in case every possible truth-value assignment that makes B true also makes A true. Semantic theorems establish that certain syntactic transformations (e.g., resolution) suffice to deduce every logical entailment of any set of premises. Other semantic theorems establish that interesting weaker syntactic deduction systems (e.g., Horn-clause resolution) are incomplete. Some truth preserving inferences escape them. Semantic theorems characterize the usually more efficiently computable subset of logical entailments that weaker deduction systems compute.

Systems for predicate logic and some modal logics also fit the ideal of syntactic deduction systems motivated by precise formal semantic theories. But many of the logics for AI systems have no rigorous semantic theory. The syntactic transformations are motivated by rough intuitions, and are adjusted to particular applications in a pragmatic but ad hoc fashion. A semantic theory furnishes a deduction system with justified *principles of correct inference*. Applications of the system turn mainly on implementing techniques for using the *principles* to make desired inferences.

This paper will present a formal semantic theory for a class of qualitative non-monotonic entailment relations. It will describe several interesting properties of these entailment relations. Then it will show how to extend the semantics to represent a class of quantitative non-monotonic partial entailment (i.e., degree of entailment) relations. These relations turn out to be the classical (i.e., Bayesian) probability functions, but with a twist. The semantics permits non-monotonic, non-Bayesian jumps from one classical probability function to another when sentences considered previously to be "impossible" (i.e., they had probability zero) are accepted as new premises.

I won't discuss syntactic deduction methods for the semantic relationships described in this paper. Nor will I prove any of the semantic theorems that establish the characteristics of the entailment relations. These theorems are proven elsewhere [3]. The purpose of this paper is to introduce an approach to a non-monotonic logic that unifies a qualitative and a quantitative notion of non-monotonic entailment into a single coherent system.

The systems in this paper will be restricted to the language of sentential logic, but the semantic theories and theorems are easily extended to a language for first-order predicate logic. Harry Field [2] first introduced a probabilistic semantics of this kind for first-order logic in 1977. That paper initiated several investigations into probabilistic semantics, including this one. For an excellent recent treatment see the papers of Bas van Fraassen [5] [6]. Other investigations are cited in its references.

The next section illustrates a typical truth-value semantics for sentential logic. This will be used in succeeding sections as a standard for comparison. Those sections will develop the semantic theory for non-monotonic entailments.

2. TRUTH

Let L be a formal language for sentential logic. L contains the following categories:

sentential letters: P_1, P_2, \dots

logical symbols: $\&, -$

parentheses: $), ($

- sentences: 1) sentential letters
 2) if A is a sentence, $\neg A$ is a sentence
 3) if A and B are sentences, then $(A \& B)$ is a sentence.

Let S be the set of all sentences of L . We use ' A ', ' B ', etc. as metalinguistic variables ranging over members of S .

The only logical symbols of L are '&' and '¬'. Other standard symbols are considered abbreviations:

- $(A \vee B)$ for $\neg(\neg A \& \neg B)$,
 $(A \rightarrow B)$ for $\neg(A \& \neg B)$,
 $(A \leftrightarrow B)$ for $(\neg(A \& \neg B) \& \neg(\neg A \& B))$.

A truth-value semantics for L specifies all possible ways the sentences of L can be simultaneously assigned truth-values -- i.e., be true or false. For sentential logic the concept of truth is the semantic primitive. Presumably the truth or falsehood of a sentence depends on the meanings of constituent terms and the actual state-of-the-world they refer to. In more complex logics (e.g., predicate logic, modal logic) the semantics may take meaning assignments and possible states-of-the-world as primitive, and define the notion of truth in terms of them. But for the purposes of sentential logic meaning and possible states add nothing essential beyond their contribution to truth. It suffices to formalize the semantics in terms of truth, and to leave meaning and world states as an informal account of how truth is determined.

A semantics for sentential logic may be specified in terms of truth-sets. Initially each subset of the set of all sentences S can be thought of as a possible truth set. Let T_a, T_b , etc. be subsets of S . If a sentence A is in T_a we say that T_a makes A true, abbreviated " $T_a(A)$ ". If A is not in T_a , then T_a makes A false, abbreviated "not $T_a(A)$ ". So each subset T_a of S can represent a possible truth-value assignment to all members of S . But not every T_a in S is a *permissible* truth-value assignment.

For T_a to be a truth-value assignment it must satisfy certain semantic rules that constrain the notion of truth:

- for all A, B in S :
- 1) $T_a(\neg A)$ iff not $T_a(A)$,
 - 2) $T_a((A \& B))$ iff $T_a(A)$ and $T_a(B)$,
- ("iff" abbreviates "if and only if").

Let TVA be the set of all T_a such that T_a is a subset of S that satisfies these semantic rules. TVA represents the set of all coherent truth-value assignments to L . Every possible meaning assignment to sentences of L together with a possible state-of-the-world corresponds to some member of TVA . And each member of TVA is a truth set for some possible meaning assignment and state-of-the-world. But the contributions of meaning and the world to truth need not be formalized in the semantics for sentential logic. The semantic rules don't require such distinctions, so they stay informally in the background.

The only notion of entailment usually associated with sentential logic is the notion of logical entailment. Logical entailment is the relation of truth preservation for all possible truth-value assignments:

definition: $A=/B$ (read "A is logically entailed by B") iff
for every T_a in TVA , if $T_a(B)$, then $T_a(A)$.

Logical entailment is both monotonic and transitive:

Monotonicity $A=/B$ only if $A=/(B\&C)$;

Transitivity $A=/B$ and $B=/C$ only if $A=/C$.

Monotonicity and transitivity are closely related in the non-monotonic entailments described in the next section.

3. ENTAILMENT

Truth-value semantics is inadequate as a basis for non-monotonic entailments because the concept of truth it explicates is monotonic to the core. Any truth-value assignment that makes A true will also make $(A\&B)$ true if B is true.

Non-monotonic logic presumes that there is more to the meaning of a sentence than the determination of truth-values at possible worlds. The meaning of a sentence (and, perhaps, the state-of-the-world) imbues a sentence with an inferential connection to other sentences. This connection is commonly expressed in one of the following ways:

- 1) If B were the case, then A would be.
- 2) A is true if B is, ceteris parabus.
- 3) B would make A nearly certain.

Each of these expressions indicates that A is entailed by B in some sense. And each expression tends to be non-monotonic. Replacing B by $(B \& C)$ can undermine the entailment. The standard example is:

1. "it flies" is entailed by "it's a bird";
2. "it flies" is not entailed by "it's a bird and it lives in the Antarctic";
3. "it doesn't fly" is not entailed by "it's a bird and it lives in the Antarctic";
4. "it flies" is entailed by "it's a bird and it lives in the Antarctic and it's a tern";
5. "it doesn't fly" is entailed by "it's a bird and it's a penguin";
6. "it's a bird" is entailed by "it's a penguin";
7. "it doesn't fly" is entailed by "it's a penguin".

The breakdown of monotonicity is illustrated by 1-5. Transitivity fails for this notion of entailment, as 1, 6, and 7 illustrate.

Truth-value semantics takes the notion of truth as primitive, and specifies truth preserving relationships. The semantics for non-monotonic entailments will take entailment as a primitive notion, and will specify entailment preserving relationships.

For language L let $S \times S$ be the set of all pairs of sentences, $\langle A, B \rangle$. Each subset of $S \times S$ is a potential entailment relation among sentences. Let $=/_a$, $=/_b$, etc. represent subsets of $S \times S$. If an ordered pair of sentences $\langle A, B \rangle$ is in $=/_a$ we say that A is entailed by B under entailment-value assignment $=/_a$, abbreviated " $A=/_a B$ ". If $\langle A, B \rangle$ is not in $=/_a$ we say that A is not entailed by B in $=/_a$, abbreviated "not $A=/_a B$ ".

Not all subsets of $S \times S$ are permissible entailment relations. The class of entailment relations of interest should satisfy certain plausible semantic rules. For each $=/_a$ in $S \times S$, $=/_a$ is in ERA (the set of permissible entailment relation assignments) just in case it satisfies the following semantic rules:

- 1) for some A and B , not $A=/_a B$;
and for all A, B, C :
- 2) $A=/_a A$;
- 3) $A=/_a (B \& C)$ only if $A=/_a (C \& B)$;
- 4.1) $(A \& B) =/_a C$ only if $(B \& A) =/_a C$;

- 4.2) $\neg(A \& B) = /_a C$ only if $\neg(B \& A) = /_a C$;
 5.1) $\neg\neg A = /_a B$ only if $A = /_a B$;
 5.2) $A = /_a B$ and $\neg A = /_a B$ only if $C = /_a B$;
 6.1) $(A \& B) = /_a C$ iff $A = /_a (B \& C)$ and $B = /_a C$;
 6.2) $\neg(A \& B) = /_a C$ iff $\neg A = /_a (B \& C)$ or $\neg B = /_a C$.

ERA characterizes a set of entailment relations. Each entailment relation assigns entailment to hold or not hold between each pair of sentences. Presumably the meanings of the sentences and the state-of-the-world contribute to the specification of an entailment relation. But for our purposes we can take the concept of an entailment relation as primitive. What is more important is that the semantic rules governing entailment relations are plausible restrictions on the intuitive notions of non-monotonic entailment we are after.

The semantic rules for *ERA* are plausible when $A = /_a B$ is read "among possible states (possible worlds) where B is true, A is almost always true". With this reading rules 1-5.2 are clearly plausible. Notice that an instance of 1 is "not $\neg B = /_a B$, for some B ". The converse of 5.1 is provable as a semantic theorem. Rule 5.2 says that if B make both A and $\neg A$ nearly certain, then B makes every sentence nearly certain. In that case we shall say that B is inconsistent in entailment relation $= /_a$.

Rule 6.1 contains a weak form of transitivity. It only permits

$$A = /_a (B \& C) \text{ and } B = /_a C \text{ only if } A = /_a C.$$

Full transitivity would say that

$$A = /_a B \text{ and } B = /_a C \text{ only if } A = /_a C.$$

Full transitivity doesn't generally hold for members of *ERA*.

The weak transitivity fits the penguin case pretty well.

$[\text{it flies}] = /_a [\text{it's a bird}]$, and
 $[\text{it's a bird}] = /_a [\text{it's a penguin}]$,
 but not $[\text{it flies}] = /_a [\text{it's a penguin}]$,
 because not $[\text{it flies}] = /_a ([\text{it's a bird}] \& [\text{it's a penguin}])$.

Rule 6.1 also permits the conjunction of entailed conjuncts. For every member $= /_a$ of *ERA*:

$$A = /_a C \text{ and } B = /_a C \text{ iff } (A \& B) = /_a C.$$

This can be proved as a semantic theorem.

Rule 6.2 contains a weak form of the deduction theorem. It implies

$$(B \rightarrow A) = /_a C \text{ only if } A = /_a (B \& C) \text{ or } \neg B = /_a C,$$

where the deduction theorem would have

$$(B \rightarrow A) = /_a C \text{ only if } A = /_a (B \& C).$$

The strong form of the deduction theorem would threaten to force monotonicity. From $A = /_a C$ we can get $(B \rightarrow A) = /_a C$, which would lead to $A = /_a (B \& C)$ with the strong version. Reading " $= /_a$ " as "is made nearly certain by", the weak version permits:

[it flies] = /_a [it's a bird],
 so ([it's a penguin] → [it flies]) = /_a [it's a bird],
 though not [it flies] = /_a ([it's a bird] & [it's a penguin]),
 because ¬[it's a penguin] = /_a [it's a bird],
 (i.e., given only that it's a bird, it almost certainly is not a penguin).

Notice that the semantics does not involve the notion of logical entailment in the truth-value sense. It is totally autonomous with respect to truth-value semantics. *ERA* semantics does not presuppose that logically equivalent sentences can be substituted in an *ERA* entailment to determine other entailments. Nor does it assume that members of *ERA* respect logical entailment. Rather, *ERA* permits an alternative definition of logical entailment.

We may define *ERA* logical entailment as entailment in every member of *ERA*:

definition: A is *ERA* logically entailed by B iff for every $= /_a$ in *ERA*, $A = /_a B$.

Semantic theorems about *ERA* show that the logical entailments in the classical *TVA* sense are just those entailments that hold in every member of *ERA*, the *ERA* logical entailments:

$A = /_a B$ iff for every $= /_a$ in *ERA*, $A = /_a B$.

So the members of *ERA* may be thought of as all possible ways of extending the classical logical entailment relation to permit additional non-monotonic entailments.

Other semantic theorems show that within each member of *ERA* one can substitute $= /_a$ -- equivalent sentences:

for all C , $A = /_a (B \& C)$ and $B = /_a (A \& C)$ only if for all D , $A = /_a D$ only if $B = /_a D$.

I.e., if A and B are monotonically equivalent in $=/_{\alpha}$ (but not necessarily logically equivalent), then whatever entails A in $=/_{\alpha}$ also entails B in $=/_{\alpha}$. The substitution rules for the premise of a relation in *ERA* doesn't require monotonic equivalence:

$$A=/_{\alpha}B \text{ and } B=/_{\alpha}A \text{ \underline{only if} for all } D, D=/_{\alpha}A \text{ only if } D=/_{\alpha}B.$$

When do monotonicity and transitivity break down for an *ERA* entailment relation? Monotonicity only fails with the addition of a new premise that was previously considered almost certainly false:

$$A=/_{\alpha}B \text{ and not } A=/_{\alpha}(B\&C) \text{ \underline{only if} } -C=/_{\alpha}B.$$

And for transitivity the following theorem holds:

$$A=/_{\alpha}B \text{ and } B=/_{\alpha}C \text{ and not } A=/_{\alpha}C \text{ \underline{only if} } -C=/_{\alpha}B.$$

Indeed, B monotonically entails A for a member $=/_{\alpha}$ of *ERA* just in case whatever entails B also entails A in $=/_{\alpha}$, i.e.:

$$\text{for every } C, A=/_{\alpha}(B\&C) \text{ iff for every } D, B=/_{\alpha}D \text{ only if } A=/_{\alpha}D.$$

The syntactic structure of *ERA* entailment relations is quite different from that of other non-monotonic logics. Other systems state explicitly in a non-monotonic inference rule what condition will counter-act the inference. In *ERA* interference with transitivity and monotonicity are signaled by other entailments that hold between the sentences involved:

$$A=/_{\alpha}B \text{ and not } A=/_{\alpha}(B\&C) \text{ can only occur when } -C=/_{\alpha}B.$$

The syntactic structure for *ERA* non-monotonic entailments resembles the non-monotonicity of conditional probabilities associated with probabilistic dependence and independence. The next section shows how closely entailments in *ERA* are related to classical probability functions.

4. PROBABILITY

The entailment semantics of the previous section extrapolates in a straightforward way to a semantics for partial (i.e., probabilistic) entailments. The class of probabilistic entailment relations turns out to be almost precisely the class of all classical (i.e., Bayesian) probability functions. They satisfy the standard axioms for probability theory -- up to a point.

A typical semantic approach to classical probability theory for a sentential language *defines* the set *PROB* of probability functions on L :

P_a is in *PROB* iff

- 1) P_a is a function from S into the real interval $[0,1]$;
- 2) $P_a(A)=1$ if A is a logical truth;
- 3) $P_a((A \vee B))=P_a(A)+P_a(B)$ if $\neg(A \& B)$ is a logical truth.

Some versions have the additional rule that logically equivalent sentences have the same probability. But that rule can be derived from 1-3 above.

On this approach probability is sometimes taken to represent the degree of certainty that a sentence is true. An alternative interpretation takes expressions like $P_a(A)=r$ to say, roughly, that given the meaning of A (that P_a presupposes), A is true in $100 \times r$ percent of the possible states-of-affairs (possible worlds).

In *PROB* semantics conditional probability is a defined notion:

definition: $P_a(A/B)=P_a((A \& B))+P_a(B)$ if $P_a(B) \neq 0$, and is undefined if $P_a(B)=0$.

Intuitively, $P_a(A/B)=r$ might be understood to say that A is true in $100 \times r$ percent of the states in which B is true.

PROB semantics relies on the concept of logical truth. Its semantic rules employ that concept. The concept of logical truth is borrowed from truth-value semantics -- i.e., truth in all members of *TVA*. Strictly speaking, the semantics employs two primitives, the concept of truth and the concept of probability.

The *ERA* semantics suggests a different approach to probabilistic semantics. For *ERA* we took certain subsets of $S \times S$ as relations which qualify as entailments. Each relation $=/a$ maps each pair of sentences onto either "entailment holds" (i.e., the pair is in $=/a$) or "entailment doesn't hold" (i.e., the pair is not in $=/a$). Let R be the set of all mappings from $S \times S$ into the real interval from 0 to 1, inclusive. Each member of R is a set of triples of form $\langle A, B, r \rangle$, where A and B are in S and r is in $[0,1]$. I will characterize a class of these functions in R that capture the notion "A is entailed by B to degree r". Members of R will be represented by symbols like $/a$, and we write " $A /_a B$ " for " $\langle A, B, r \rangle \in /a$ ". $/a$ is a mapping, i.e., a function, so $\langle A, B, r \rangle \in /a$ and $\langle A, B, s \rangle \in /a$ only if $r=s$ (r and s reals in $[0,1]$). R contains all and only such functions. Every sentence is entailed by each sentence to some unique degree for each partial entailment function $/a$ in R .

Define *PVA* as the set of all probabilistic entailment value assignments $/a \in R$ that meet the following conditions:

- 1) for some $A, B \in S$, not $A /_a B$;
and for every $A, B, C \in S, r, s, q \in [0,1]$:

- 2) $A \downarrow_a A$;
- 3) $A \downarrow_a (B \& C)$ only if $A \downarrow_a (C \& B)$;
- 4) $(A \& B) \downarrow_a C$ only if $(B \& A) \downarrow_a C$;
- 5) $A \downarrow_a B$ and $\neg A \downarrow_a B$ and $r+s \neq 1$ only if $C \downarrow_a B$;
- 6) $(A \& B) \downarrow_a C$ and $A \downarrow_a (B \& C)$ and $B \downarrow_a C$ only if $r=s \times q$.

Each rule is the obvious extension of a similarly numbered rule for *ERA*. Rule 5 is the natural extension of 5.2, and covers 5.1. Rules 4 and 6 extend 4.1 and 6.1. The connection between a sentence and its negation imposed by Rule 5 is sufficiently strong to cover the counterparts to 4.2 and 6.2.

Since \downarrow_a is a function we can establish a notational convenience. We will rewrite $A \downarrow_a B$ as $P_a(A/B)=r$, and say $P_a \in PVA$ rather than $\downarrow_a \in PVA$. Rewriting 1 through 6 we have:

- 1) for some $A, B \in S$, $P_a(A/B) \neq 1$;
and for all $A, B, C \in S$:
- 2) $P_a(A/A)=1$;
- 3) $P_a(A/(B \& C))=P_a(A/(C \& B))$;
- 4) $P_a((A \& B)/C)=P_a((B \& A)/C)$;
- 5) $P_a(A/B)+P_a(\neg A/B)=1$ or $P_a(C/B)=1$;
- 6) $P_a((A \& B)/C)=P_a(A/(B \& C)) \times P_a(B/C)$.

Notice that it is not *assumed* that the members of *PVA* are classical probability functions, nor that logical entailments have conditional entailments of 1. *PVA* contains just those P_a that are functions from $S \times S$ into $[0, 1]$ satisfying conditions 1-6.

Presumably, sentences are true or false because of their meanings and the state-of-the-world. *TVA* doesn't make such distinctions because they contribute nothing essential to sentential logic. Only when the formal language is extended to intentional contexts (e.g., modal operators) need the semantics explicitly reflect the separate contributions to truth by meaning and the possible world or state-of-affairs that an interpretation takes the sentences to be about.

Similarly, the semantics for partial entailments given by *PVA* need not make explicit the separate contributions made by meaning and the nature of possible states. Presumably, $A \downarrow_a B$ holds under interpretation \downarrow_a because \downarrow_a represents both a way of associating meaning with A and B , and certain probabilistic relationships among the possible states-of-affairs that A and B are about. Roughly, $A \downarrow_a B$ says that r is the measure or frequency of A being true among possible states (possible worlds) where B is true. Parsing members of *PVA* into these components may play an essential role in a semantics for an intentional language. But, for our purposes it is only a useful heuristic for understanding what partial entailment represents. Understood in this way, the semantical rules are plausible principles for partial entailments.

Semantic Rules 1-4 seem totally uncontroversial. Rule 5 is plausible, too. Presumably each possible state makes either A true or $\neg A$ true. So their truth-frequencies should add to I among states where B is true. Rule 6 is also a plausible principle when read in terms of truth-frequencies among possible states. Of course any other interpretation of partial entailments (e.g., as conditional degrees of belief) is also captured by *PVA* provided it satisfies the semantic rules.

PVA semantics does not presuppose a notion of logical entailment. Like *ERA*, it permits an independent *definition* of logical entailment:

A is *PVA* logically entailed by B iff for all P_a in *PVA*, $P_a(A/B)=I$.

Then a semantic theorem establishes that the defined notion coincides with classical *TVA* logical entailment. Logical truth is just logical entailment by every sentence. So that notion, too, is definable in *PVA* semantics.

Relative to any given sentence C , for each P_a in *PVA* the function $P_a(/C)$ satisfies the classical probability rules of *PROB*. This is provable as a semantic theorem. Observe that Rule 6 for *PVA* requires that conditionalization relative to a sentence B fits the classical definition of conditional probability when $P_a(B/C)\neq 0$:

$$P_a(A/(B\&C))=P_a((A\&B)/C)+P_a(B/C) \text{ if } P_a(B/C)\neq 0.$$

So $P_a(/C)$ behaves as classical probability unless conditionalized on a sentence B that is "nearly impossible", has measure 0 , relative to C . But entailment to degree 0 need not make B absolutely impossible. Rule 6 permits $P_a(/(B\&C))$ to behave as a new classical probability function. It, too, behaves classically until some new condition D for which $P_a(D/(B\&C))=0$ is added. Then $P_a(/(D\&(B\&C)))$ behaves classically, and so on.

Bayes' theorem is a direct consequence of classical conditionalization:

$$P_a(A/(B\&C))=P_a(A/C)\times[P_a(B/(A\&C))+P_a(B/C)] \text{ for } P_a(B/C)\neq 0.$$

It states how adding a new premise B influences the degree to which A is entailed by C . Rule 6 requires members of *PVA* to transform in classical Bayesian fashion when $P_a(B/C)\neq 0$. When $P_a(B/C)=0$, Rule 6 permits $P_a(A/(B\&C))$ to take a non-Bayesian leap -- $P_a(/(B\&C))$ becomes a different classical probability function than $P_a(/C)$, and they are not related by a Bayesian transformation. In effect each member of *PVA* is a class of Bayesian probability functions with non-Bayesian jumps from one to another impelled by new conditions considered nearly impossible under previous conditions.

5. CONCLUSION

Probabilistic inference is itself non-monotonic, and some have suggested it subsumes the qualitative notion. Cheeseman [1] and Heurion [4] have argued extensively that classical probability theory is sufficient for all forms of reasoning under uncertainty. They argue that the numerous other theories for uncertain inference developed for AI applications suffer disorders ranging from being simply unnecessary (i.e., classical probability would do as well), to ad hoc and misleading, to unsound. While I largely share their views regarding quantitative alternatives to classical probability, I take a different view of qualitative non-monotonic inference.

The semantic theories described above show that classical probability theory is a simple quantitative extension of an underlying semantic theory of non-monotonic entailments. Entailments are not subsumable under the Bayesian conditionalization mechanism of classical probability. Rather, non-monotonic entailments can furnish non-Bayesian jumps from one classical probability function to another. The semantic theory suggests a general approach to uncertain inference that unifies qualitative and quantitative non-monotonic inference into a single coherent system.

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